GENERAL MAGNETIC FIELD OF THE SUN — BASED ON MAGNETOGRAMS I

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(Received April 3, 1959)

The magnetograms made in the Mount Wilson Observatory, the present of H. W. BABKOCK, are investigated. In the first part the mathematical background of the suggested statistical method is summarized, a practical method is presented, furthermore, a numerical prototype is investigated and some preliminary conclusions are suggested.

§ 1. *Introduction*

As is well known the magnetograms show the components of the photospheric magnetic field in the line of sight consisting of the superposition of the general and random field, respectively. The average of the latter vanishes, therefore, the mean value of the photospheric field determines the general field. A suitable way for the determination of the mean values seems to be the calculation of the integral of the magnetic curves along the horizontal traces across the solar disc on a magnetogram from the west limb to the east one. These integrals give, however, only the average of the general field in relation to these lines, therefore, the determination of the characteristics of the general field can be explained only by further analysis of the magnetograms.

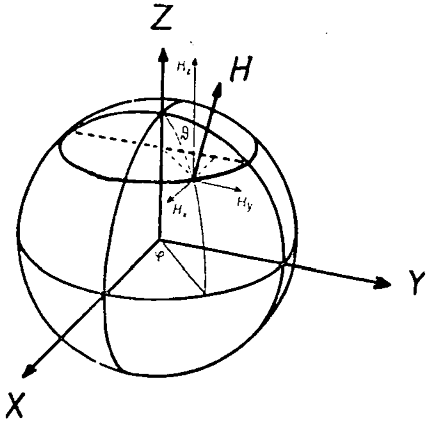

Fig. 1

§ 2. *The components of the general field in the line of sight*

1. Let the general magnetic field be represented by the scalar potential as follows

$$\Phi = \sum \frac{n h_n P_n(\cos \vartheta)}{r^{n+1}}. \qquad (1)$$

A system of axes X, Y, Z may be introduced (see Fig. 1) in such a way that the X axis is oriented toward the observer (to Earth), Z to north along the central meridian, and the axis Y to east. The components of the magnetic field in this system are:

$$\mathfrak{H}_x = -\frac{\partial \Phi}{\partial r} \sin \vartheta \cos \varphi - \frac{1}{r} \frac{\partial \Phi}{\partial \vartheta} \cos \vartheta \cos \varphi$$

$$\mathfrak{H}_y = -\frac{\partial \Phi}{\partial r} \sin \vartheta \sin \varphi - \frac{1}{r} \frac{\partial \Phi}{\partial \vartheta} \cos \vartheta \sin \varphi$$

$$\mathfrak{H}_z = -\frac{\partial \Phi}{\partial r} \cos \vartheta + \frac{1}{r} \frac{\partial \Phi}{\partial \vartheta} \sin \vartheta$$

Applying (1) and using some recurrence formulae [1] the expansions

$$\mathfrak{H}_x = \sum \frac{n h_n}{r^{n+2}} P_{n+1}^{(1)} \cos \varphi, \quad \mathfrak{H}_y = \sum \frac{n h_n}{r^{n+2}} P_{n+1}^{(1)} \sin \varphi, \quad \mathfrak{H}_z = \sum \frac{n(n+1) h_n}{r^{n+2}} P_{n+1} \quad (2)$$

can be obtained.

In the magnetograms the horizontal lines correspond to the orthogonal projections of the circle $\varphi = const.$ on the XZ plane. The averages along these circles correspond to an integration of the traces between the limits $-\pi/2$ and $+\pi/2$ divided by the length of the zero-line. This means that

$$\bar{\mathfrak{H}} = \frac{1}{\pi r_{\odot} \sin \vartheta} \int_{-\pi/2}^{+\pi/2} \mathfrak{H} r_{\odot} \sin \vartheta \, d\varphi.$$

Taking the average of (2) after some calculations one obtains simply that

$$\bar{\mathfrak{H}}_x = \frac{2}{\pi} \sum \frac{n h_n}{r^{n+2}} P_{n+1}^{(1)}, \quad \bar{\mathfrak{H}}_y = 0, \quad \bar{\mathfrak{H}}_z = \frac{2}{\pi} \sum \frac{n(n+1) h_n}{r^{n+1}} P_{n+1}. \quad (3)$$

In the following we have only to take component $\bar{\mathfrak{H}}_x$ into account as it is the observed one. If on the two hemispheres the magnetic properties are opposite the odd terms of (1) vanish and the remaining first two terms of $\bar{\mathfrak{H}}_x$ will be:

$$\frac{\pi}{2} \bar{\mathfrak{H}}_x = \frac{h_1}{r^3} P_2^{(1)} (\cos \vartheta) + \frac{3 h_3}{r^5} P_4^{(1)} (\cos \vartheta)$$

or introducing the nondimensional parameter $h = h_3/(h_1 r_{\odot}^2)$

$$\frac{\pi}{2} \bar{\mathfrak{H}}_x = \frac{h_1}{r^3} \left[P_2^{(1)} (\cos \vartheta) + 3h \frac{r_{\odot}^2}{r^2} P_4^{(1)} (\cos \vartheta) \right] \quad (4)$$

can be obtained which will be the fundamental expression for the practical interpretation.

2. In the above it was implicitly assumed that the magnetic axis coincides with the axis Z. In the case of a more general discussion one has also to take the annual deviations of the solar axis from the axis Z into account.

One can substitute $\cos\vartheta$ by $\cos\vartheta'$ for which

$$\cos\vartheta' = \cos\vartheta\cos q + \sin\vartheta\sin q\cos(\varphi - \Phi)$$

holds, where q and Φ are the polar coordinates of the north solar pole in the system X, Y, Z.

Owing to the additional theorem of the Legendre functions [2]

$$P_n(\cos\vartheta') = P_n(\cos\vartheta)P_n(\cos q) + $$
$$+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\vartheta)P_n^m(\cos q)\cos m(\varphi-\Phi),$$

it can be concluded that $\mathfrak{H}_x$ — in this more general form — depends also on the coordinate φ. The x-component of the magnetic strength, will in this case be completed by a term resulting from derivatives of φ, *i. e.*

$$\mathfrak{H}_x = -\frac{\partial\Phi}{\partial r}\sin\vartheta\cos\varphi - \frac{1}{r}\frac{\partial\Phi}{\partial\vartheta}\cos\vartheta\cos\varphi - \frac{1}{r\sin\vartheta}\frac{\partial\Phi}{\partial\varphi}\sin\varphi. \qquad (5)$$

At the deduction of the mean values along the horizontal lines given by $\varphi = const.$, one meets the following integrals

$$\int_{-\pi/2}^{+\pi/2}\cos m(\varphi-\Phi)\cos\varphi\, d\varphi = \begin{cases}\pi/2\cdot\cos\Phi & m=1\\ 0 & m<1\end{cases}$$

and

$$\int_{-\pi/2}^{+\pi/2}\cos m(\varphi-\Phi)\sin\varphi\, d\varphi = \begin{cases}\pi/2\cdot\sin\Phi & m=1\\ 0 & m>1\end{cases}.$$

It is a general rule of magnetograms that the horizontal lines are perpendicular to the central meridian, *i. e.* $\Phi = 0$. This means that the second integral identically vanishes.

The expansion of $\bar{\mathfrak{H}}_x$ in this more general case is rather complicated but according to the following discussion it can be separated as follows:

$$\bar{\mathfrak{H}}_x = \sum\frac{2nP_{n+1}^{(1)}(\cos\vartheta)}{r^{n+2}}P_n(\cos q) + F(\cos\vartheta), \qquad (6)$$

where $F(\cos\vartheta)$ denotes a function being symmetrical to the equator ($\vartheta = 90°$) for odd n. The first term correspond to the x-component of (3), but has coefficients $P_n(\cos q)$ depending on the position of the solar pole. The second can be eliminated by substraction of the mean values belonging to the horizontal lines given by $\vartheta(<90°)$ and $180° - \vartheta$. The expression

$$H_x = \frac{\bar{\mathfrak{H}}_{xN} - \bar{\mathfrak{H}}_{xS}}{2} = \sum\frac{2nh_n}{r^{n+2}}P_n(\cos q)P_n^{(1)}(\cos\vartheta) \qquad (7)$$

gives the final form for the practical calculation, where $\bar{\mathfrak{H}}_{xN}$ and $\bar{\mathfrak{H}}_{xS}$ denote the values of $\overline{\mathfrak{H}}_x$ at ϑ and at $180° - \vartheta$, respectively. In the case of the two-terms potential function the formula

$$H_x = \frac{2h_1}{r^3} P_1(\cos q)\left[P_1^{(1)}(\cos\vartheta) + 3h\frac{P_3(\cos q)}{P_1(\cos q)}\frac{r_\odot^2}{r^2}P_4^{(1)}(\cos\vartheta)\right] \qquad (8)$$

can be obtained, deviating from the fundamental expression (4) by the coefficient of $P_3(\cos q)/P_1(\cos q)$ corresponding to an annual correction of h.

The order of magnitude of this correction may be estimated when the maximum of q is substituted in the explicit form of the coefficient:

$$P_3(\cos q)/P_1(\cos q) = (1/2)(5\cos^2 q - 3)$$

providing a numerical factor of 0,96 (being $q \sim 7°$ in maximum) which means an additional correction of about 4 *per cent.*

3. A further correction may arise from the effects of the limb-darkening which depends on the position p relative to the centre of the disc in the following form:

$$\mathfrak{H}_x' = \frac{2}{5}\mathfrak{H}_x\left(1 + \frac{3}{2}\cos p\right)$$

where $\mathfrak{H}'$ means the registered value. Owing to the formula $\cos p = \sin\vartheta\cos\varphi$, finally

$$\overline{\mathfrak{H}}_x' = \frac{2}{5}\overline{\mathfrak{H}}_x + \frac{3}{5}\sin\vartheta\,\overline{\mathfrak{H}_x\cos\varphi}$$

can be obtained.

The last term of the right hand side leads to an expression like (6) and it can be eliminated in a similar way.

§ 3. *A practical method for the calculation of averages introduced*

The determination of $\overline{\mathfrak{H}}_x$ can be directly realized by means of a planimeter (2), the calibration of which can be carried out as follows: an area of 5 mm^2 corresponds to 0,1 in the scale which is the lower limit of the measurements, but a copy of 12 cm in diameter from the microfilm of the magnetograms is also suitable (the value of the integral in these cases was between 0,5 an 1,0 in the scale). The repetition of the mesurement has been given a spread of 0,2—0,3 proving the reality of the method suggested.

The calibration in gauss was not performed as the nondimensional parameter h was discussed which is independent of calibration.

According to the expression (7) a condition of symmetry has to be fulfilled by the horizontal lines. An exact elimination of $F(\cos\vartheta)$ is possible only in the case when the traces are lying symmetrical to the centre. However, this condition is not generally realized, therefore the procedure (7) has a systematic error, the approximate values of which are given by

$$F(\cos\vartheta_1) - F(\cos\vartheta_2) = \frac{\partial F(\cos\vartheta)}{\partial\vartheta}d\vartheta.$$

Because the derivative is of unit in order, the left hand side depends only on $d\vartheta$. An estimated value of $d\vartheta$ may be about 2°, consequently the assimetry provides an additional term of order of 4 *per cent*. This affects the averages very little and together with the annual variation of h they will not be taken into account in the following discussions.

§ 4. *A numerical illustration of the method*

In order to use the method elaborated above, three different kinds of measurements have to be carried out for the evaluation of the discs: (i) determination of the areas by planimeter, (ii) measurement of the lengths of the horizontal lines and (iii) its heliographical latitudes, respectively. Table I shows these numerical values corresponding to a disc. The coefficient in the last column means the factor of calibration composed by the calibration of magnetograms and that of the planimeter.

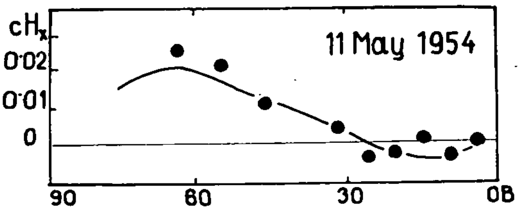

Fig. 2

Owing to the numerical values the coefficients A and B of the formula of interpolation

$$cH_x = AP_2^{(1)} + BP_4^{(1)}$$

are calculated by the method of the least squares, *e.g.* using the numerical values given in Table I., it will be got: $A = 0{,}00716$, $B = 0{,}00465$. According to the definition of h one can easily obtain that $h = B/(3A) = 0{,}216$. Fig. 2 shows a comparison of the measured values of cH_x and its calculated curve. The dots indicate the values of $1/2\,[H_x(\vartheta) - H_x(180° - \vartheta)]$.

Some magnetograms give unacceptable large value for h, illustrated by Fig. 3. One can immediately see that the formula of interpolation is not suitable for these observations and the conclusion can be drawn that the

Table I

11 May 1954

n	Pl	d	B	cH_x	n	Pl	d	B	cH_x
1	1,0	57	66	0,032	11	0,8	120	— 3	0,007
2	1,6	71	55	0,024	12	1,4	118	— 9	0,012
3	1,3	83	46	0,016	13	1,4	116	—14	0,012
4	0,5	95	38	0,005	14	1,3	113	—20	0,012
5	1,0	103	31	0,013	15	1,9	108	—26	0,018
6	1,4	109	24	0,013	16	0,5	102	—32	0,005
7	1,3	114	19	0,011	17	0,5	94	—38	0,005
8	1,8	117	13	0,015	18	—0,7	83	—46	—0,008
9	0,8	118	7	0,007	19	—1,3	69	—54	—0,015
10	0,8	120	2	0,007	20	—1,0	42	—62	—0,020

Pl = areas determined by planimeter, d = length of horizontal lines, B = heliographical latitude of the lines, cH_x = magnetic strength with calibration factor.

solar magnetic field would not be characterized by (4) in this case. It seems that in this case the usual magnetic field of the Sun would be deformed by an extended unipolar field discovered by H. W. BABCOCK [3]. These magnetograms, however, may be omitted from our point of wiew as they are incompatible with a statistical evalution of h.

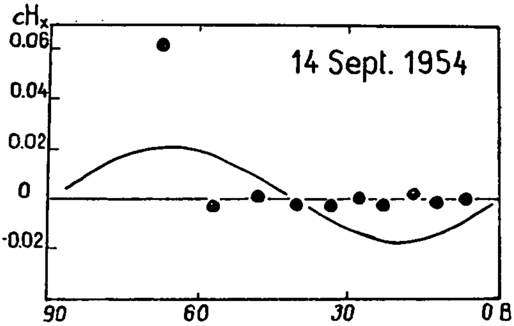

Fig. 3

§ 5. *Results and conclusions*

Table II contains the values of h calculated from 99 magnetograms by the suggested method. The magnetograms were obtained from H. W. BABCOCK as microfilms. A second series of these magnetograms will be published in the next future.

The conclusions of the present article can be summarized as follows:

(a) Most of the magnetograms permit for h values between 0 and 1. But there are some magnetograms with negative values of h between 0 and —0,5, taken since Oct. 1, 1954.

(b) Some magnetograms give imcompatible values for h. In five cases h has a greater value that 1 and in seven cases it is smaller than —0,5. The number of the accepted magnetograms are 87 indicated in Fig. 4 by dots and the rest by circles.

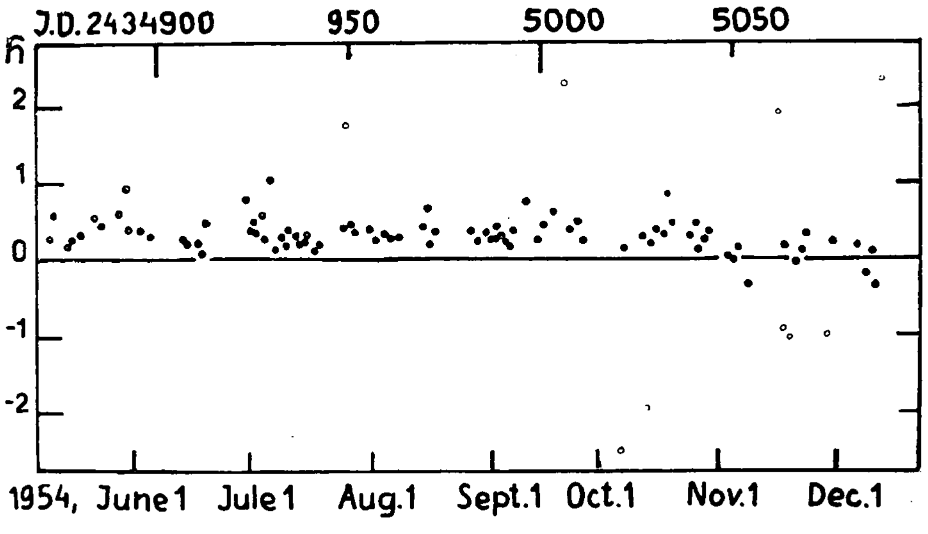

Fig. 4

(c) It seems to be very probable that h has a periodical variation of 30 days. However, the present observational material is from the statistical point of view not large enough to draw such a conclusion safely. In the second part of this paper we will have the possibility to investigate a greater observational material and it is hoped that it will be possible to return to this problem. If this periodical variation would be proved it might be concluded that the magnetic axis does not coincide with the rotational one. It can be mentioned that this conclusion would be accepted as the period of 30 days is in good agreement with the rotational period of the photosphere at the polar areas.

(d) The only conclusion which may be obtained from the present material is that the average of h is about 0,4 being in good agreement with a pre-

Table II

Date		J. D.	h
1954		2434	
May	11.	874	0,216
	12.	875	0,551
	16.	879	0,134
	17.	880	0,230
	18.	881	(1,171)
	19.	882	0,297
	23.	886	0,508
	24.	887	0,416
	31.	894	0,919
June	1.	895	0,362
	2.	896	(—18,266)
	6.	900	0,252
	14.	908	0,237
	15.	909	0,223
	18.	912	0,160
	19.	913	0,0965
	20.	914	0,476
	30.	924	0,788
July	1.	925	(1,579)
	2.	926	0,361
	3.	927	0,498
	4.	928	0,317
	5.	929	0,540
	6.	930	0,259
	7.	931	1,022
	9.	933	0,136
	10.	934	0,277
	11.	935	0,189
	12.	936	0,386
	14.	938	0,286
	15.	939	0,180
	16.	940	0,221
	17.	941	0.293

Date		J. D.	h
1954		2434	
July	19.	943	0,120
	20.	944	0,199
	26.	950	0,418
	27.	951	(1,750)
	28.	952	0,449
	29.	953	0,360
Aug.	2.	957	0,368
	4.	959	0,228
	6.	961	0,308
	8.	963	0,259
	10.	965	0,295
	16.	971	0,433
	17.	972	0,690
	18.	973	0,181
	20.	975	0,347
	28.	983	0,374
	31.	986	0,238
Sept.	2.	988	0,346
	3.	989	0,254
	4.	990	0,260
	5.	991	0,406
	6.	992	0,289
	7.	993	0,225
	8.	994	0,216
	9.	995	0,355
	13.	999	0,723
	14.	5000	(—9,075)
	16.	002	0,218
	17.	003	0,433
	20.	006	0,387
	22.	008	(2,333)
	24.	010	0,375
	25.	011	0,485

Date		J. D.	h
1954		2435	
Sept.	27.	013	0,238
Oct.	7.	023	(—2,521)
	8.	024	0,213
	13.	029	0,271
	14.	030	(—1,950)
	15.	031	0,188
	16.	032	0,362
	18.	034	0,335
	19.	035	0,857
	20.	036	0,495
	25.	041	0,271
	26.	042	0,448
	27.	043	0,136
	28.	044	0,206
	29.	045	0,348
Nov.	4.	051	0,0388
	5.	052	—0,0154
	6.	053	0,150
	9.	056	—0,341
	17.	064	(1,971)
	18.	065	(—0,901)
	19.	066	0,196
	20.	067	(—2,018)
	22.	069	—0,0252
	23.	070	0,140
	24.	071	0,366
	30.	077	(—0,976)
Dec.	1.	078	0,242
	8.	085	0,181
	10.	087	—0.214
	11.	088	0,0944
	12.	089	—0,348
	15.	092	(2,390)

vious result of the author [1], where h was derived from the curvature of the polar rays of solar corona and numerical values of the same order of magnitude was obtained.

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Sincere gratitude is expressed to DR. H. W. BABKOCK for sending some magnetograms of the Mount Wilson Observatory.

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